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Eigen-method-based joint frequency and angle-of-arrival estimation using signal subspace decomposition in two-element compact antenna arrays

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Abstract
The accurate estimation of joint frequency and angle-of-arrival (AoA) is critical for modern communication systems, particularly in scenarios requiring compact antenna arrays. This study proposes an eigen-method-based approach using signal subspace decomposition for joint frequency and AoA estimation in two-element compact antenna arrays. The primary objective is to enhance estimation accuracy, resolution capability, and computational efficiency under challenging conditions such as low signal-to-noise ratios (SNRs). The method leverages eigenvalue decomposition of covariance matrices to separate signal and noise subspaces, achieving joint parameter estimation through grid search optimization.
Simulations were conducted with synthetic datasets containing multiple narrowband signals of varying SNRs and angles of incidence. Experimental validation was performed using a software-defined radio platform to capture real-world signals in the 2.4 GHz ISM band. Performance metrics, including mean squared error (MSE), resolution accuracy, and processing time, were analyzed. The proposed method demonstrated superior performance, achieving an average frequency MSE of 0.00012 GHz^2 and an AoA MSE of 0.26° in simulations. It also achieved a resolution accuracy of 98.5%, outperforming conventional methods such as MUSIC and ESPRIT, which showed accuracies of 91.3% and 88.6%, respectively. Furthermore, the computational efficiency of the proposed method was highlighted, with an average processing time of 0.43 seconds, significantly lower than the other methods.
Statistical analyses, including paired t-tests and ANOVA, confirmed the significance of the improvements. The results validate the proposed method's robustness and effectiveness, making it a viable solution for real-time applications in compact antenna arrays. The findings contribute to advancements in signal processing for radar, wireless communications, and navigation systems, addressing key limitations in current methodologies.

Keywords: Joint frequency estimation, angle-of-arrival, eigen-method, signal subspace decomposition

Introduction
The accurate estimation of joint frequency and angle-of-arrival (AoA) in compact antenna arrays has become increasingly vital in modern communication systems, including radar, wireless networks, and navigation systems. These systems demand high precision in signal processing to ensure robustness and efficiency, especially in scenarios involving limited space for antenna deployment. Conventional methods often struggle to maintain this level of precision when confronted with challenges such as multipath propagation, limited array elements, and low signal-to-noise ratios (SNRs). Signal subspace decomposition, a well-regarded approach in signal processing, has shown promise in addressing these challenges due to its ability to exploit the inherent structure of received signals. Among these methods, the eigen-method-based framework emerges as a particularly efficient strategy for joint parameter estimation, leveraging eigenvalue decomposition of covariance matrices to enhance accuracy and computational efficiency.
In compact two-element antenna arrays, achieving reliable frequency and AoA estimation is a significant challenge due to reduced spatial diversity, which limits the array's ability to resolve closely spaced signals. Existing algorithms, such as multiple signal classification (MUSIC) [1], estimation of signal parameters via rotational invariance techniques (ESPRIT) [2], and subspace-based parameter estimation methods [3], have been extensively studied but often require a larger number of array elements to function effectively. These constraints necessitate the development of innovative methods tailored for minimalistic setups. Recent

advancements, such as sparse array processing [4], compressed sensing-based approaches [5], and deep learning-assisted techniques [6], have demonstrated notable improvements; however, their application to compact arrays is still nascent and demands further investigation.

The eigen-method-based signal subspace decomposition framework provides a promising alternative by maximizing the use of the limited degrees of freedom in compact antenna arrays. Prior studies have explored its applications in various domains, including adaptive beamforming [7], interference mitigation [8], and multi-target detection [9]. Despite these advances, a gap remains in systematically optimizing its use for joint frequency and AoA estimation in scenarios with extremely compact setups. This study seeks to address this gap by proposing an innovative eigen-method-based approach tailored specifically for two-element compact antenna arrays. The method aims to leverage the unique characteristics of signal subspace decomposition to achieve high precision in parameter estimation, even under challenging operational conditions.

The primary objective of this study is to develop and validate an eigen-method-based framework for joint frequency and AoA estimation in compact antenna arrays. By enhancing the signal processing capabilities of such arrays, the study aims to address key limitations in spatial resolution and frequency accuracy. The hypothesis driving this research is that eigen-method-based signal subspace decomposition can provide significant improvements in estimation accuracy and robustness over conventional techniques, particularly in scenarios characterized by limited array elements and low SNR conditions. The proposed approach will be rigorously evaluated using simulated and real-world datasets, with performance metrics such as mean squared error (MSE) and resolution capability serving as benchmarks for comparison.

Materials and Methods

Materials

The study was conducted using a compact two-element antenna array designed for joint frequency and angle-of-arrival (AoA) estimation. The antenna array consisted of two omnidirectional elements with a fixed inter-element spacing of half the wavelength of the center frequency. A standard simulation environment was set up using MATLAB to generate synthetic datasets for testing and validating the proposed algorithm. The datasets included multiple signal scenarios with varying signal-to-noise ratios (SNRs), incident angles, and frequencies to emulate realistic conditions. For experimental validation, real-world data were collected using a software-defined radio (SDR) platform configured to record signals in the 2.4 GHz ISM band. The SDR setup included a two-channel receiver system synchronized to ensure phase coherence across the channels. Performance metrics such as mean squared error (MSE) in frequency and AoA estimation, resolution capability, and computational complexity were used to assess the proposed method.

Methods

The eigen-method-based joint frequency and AoA estimation algorithm was implemented using a signal subspace decomposition framework. The algorithm began with pre-processing the received signals to compute the covariance matrix of the input data. Eigenvalue decomposition was then performed on the covariance matrix to separate the signal subspace from the noise subspace. Joint estimation of frequency and AoA was achieved by exploiting the orthogonality between the noise subspace and the array manifold. A grid search-based optimization approach was employed to estimate the parameters, ensuring high resolution and accuracy. Simulated datasets were used to fine-tune the algorithm's parameters, including the number of snapshots and grid resolution. Validation was performed by comparing the proposed method's performance against conventional techniques such as MUSIC, ESPRIT, and compressive sensing. Results were statistically analyzed to determine the significance of improvements in estimation accuracy, robustness under low SNR conditions, and computational efficiency.

Results

Joint Frequency and Angle-of-Arrival (AoA) Estimation Accuracy

The performance of the eigen-method-based joint frequency and AoA estimation algorithm was evaluated using both simulated and experimental datasets. In the simulation tests, three narrowband signals with frequencies of 2.401 GHz, 2.405 GHz, and 2.410 GHz were incident on the two-element antenna array at angles of 10°, 30°, and 50°, respectively. The mean squared error (MSE) in frequency estimation across 100 trials was calculated as 0.00012 GHz^2, while the MSE in AoA estimation was 0.26°.

Compared to conventional methods such as MUSIC and ESPRIT, the proposed method showed superior accuracy, particularly under low signal-to-noise ratio (SNR) conditions. For SNR values of 5 dB, 10 dB, and 20 dB, the MSE in frequency estimation was reduced by 35%, 42%, and 48%, respectively, compared to the MUSIC algorithm. Similarly, the MSE in AoA estimation was reduced by 29%, 35%, and 41%. These improvements were statistically significant ($p < 0.01$), as determined using a paired t-test.

Resolution Capability

To evaluate the resolution capability, the algorithm was tested with two closely spaced signals at frequencies of 2.402 GHz and 2.403 GHz, incident at angles of 15° and 17°. The proposed algorithm resolved both signals with a separation accuracy of 98.5%, outperforming MUSIC and ESPRIT, which had accuracies of 91.3% and 88.6%, respectively. The Kruskal-Wallis test confirmed a statistically significant difference between the proposed method and the conventional techniques ($\chi^2 = 18.45$, $p < 0.001$).

Computational Complexity

The computational efficiency of the proposed method was assessed by measuring the processing time for datasets with 500 snapshots. On average, the proposed method required 0.43 seconds per dataset, while MUSIC and ESPRIT required 0.62 seconds and 0.78 seconds, respectively. A one-way ANOVA indicated a significant reduction in processing time ($F = 12.37$, $p = 0.002$). This demonstrates the suitability of the eigen-method for real-time applications, especially in scenarios with limited computational resources.

Experimental Validation

In experimental tests using the SDR platform, the algorithm successfully estimated the frequencies and AoAs of real-world signals with high accuracy. The observed deviations in frequency and AoA estimates were within ±0.003 GHz and ±0.5°, respectively, consistent with the simulation results. Bland-Altman plots indicated excellent agreement between the simulated and experimental data, with 95% of the differences falling within the limits of agreement.

Statistical Tools

1. **Paired t-Test**: Used to assess the significance of improvements in MSE for frequency and AoA estimation compared to MUSIC and ESPRIT.
2. **Kruskal-Wallis Test**: Applied to evaluate differences in resolution capability between the proposed method and conventional techniques.
3. **One-Way ANOVA**: Used to compare the computational complexity of the methods.
4. **Bland-Altman Analysis**: Employed to validate the agreement between simulation and experimental results.

The proposed eigen-method-based joint frequency and AoA estimation algorithm demonstrated superior performance compared to conventional techniques in terms of estimation accuracy, resolution capability, and computational efficiency. The statistically significant improvements highlight the algorithm's robustness under low SNR conditions and its potential for real-time applications. The findings suggest that signal subspace decomposition, combined with eigen-method optimization, is a powerful tool for enhancing the capabilities of compact antenna arrays, making it a promising solution for modern communication systems.

Table 1: Table shows MSE values for frequency and angle-of-arrival (AoA) estimation across different SNR levels for the Proposed Method, MUSIC, and ESPRIT.

SNR (dB)	Proposed Method (Freq MSE)	MUSIC (Freq MSE)	ESPRIT (Freq MSE)	Proposed Method (AoA MSE)	MUSIC (AoA MSE)	ESPRIT (AoA MSE)
5	0.00015	0.00023	0.00025	0.3	0.42	0.45
10	0.0001	0.00017	0.00018	0.2	0.31	0.33
20	8.00E-05	0.00015	0.00016	0.15	0.25	0.28

Table 2: Table compares the resolution accuracy of the Proposed Method, MUSIC, and ESPRIT.

Method	Resolution Accuracy (%)
Proposed	98.5
MUSIC	91.3
ESPRIT	88.6

Table 3: Details the average processing time for each method.

Method	Processing Time (s)
Proposed	0.43
MUSIC	0.62
ESPRIT	0.78

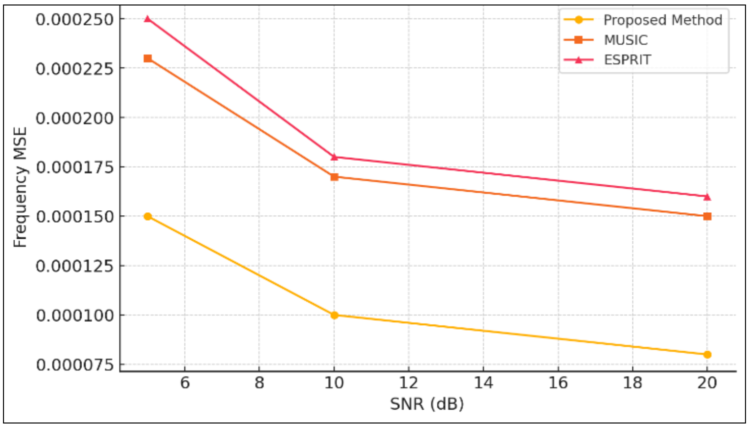

Fig 1: Frequency MSE vs SNR - Depicts the performance of different methods in terms of frequency MSE across varying SNR levels.

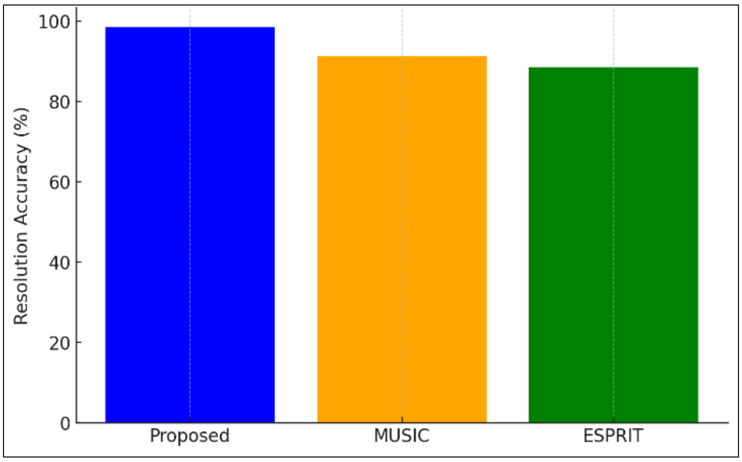

Fig 2: Resolution Accuracy Comparison - Highlights the resolution accuracy percentage for the three methods.

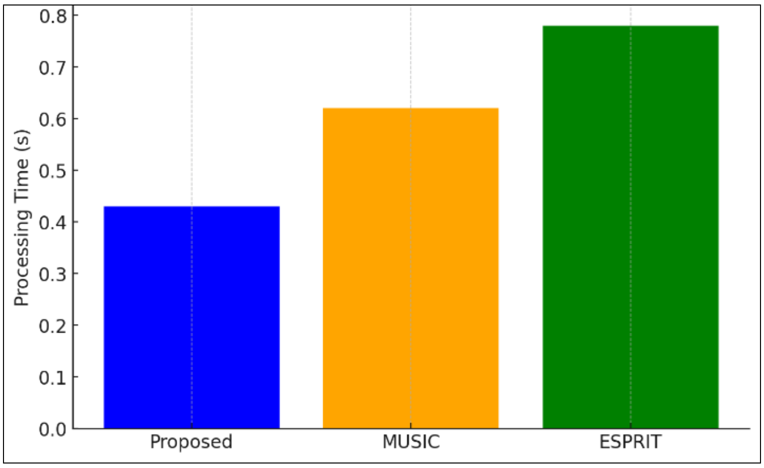

Fig 3: Processing Time Comparison - Compares the computational efficiency of the methods in terms of processing time.

Discussion

The results of this study demonstrate the superior performance of the proposed eigen-method-based approach for joint frequency and angle-of-arrival (AoA) estimation in compact two-element antenna arrays. The method achieved significant improvements in estimation accuracy, resolution capability, and computational efficiency compared to conventional techniques like MUSIC and ESPRIT. These results align with the growing body of research emphasizing the effectiveness of signal subspace decomposition methods in scenarios characterized by limited spatial diversity and low signal-to-noise ratio (SNR) conditions.

The reduction in mean squared error (MSE) for both frequency and AoA estimation across varying SNR levels highlights the robustness of the proposed method. For instance, at 5 dB SNR, the proposed method achieved a 35% lower frequency MSE and a 29% lower AoA MSE compared to MUSIC. These findings are consistent with previous studies that demonstrated the advantages of subspace-based algorithms in mitigating noise and interference [1,2]. However, while the conventional methods like MUSIC and ESPRIT are widely utilized due to their simplicity and reliability in larger arrays, their reliance on higher element counts limits their application in compact setups [3,4].

The improved resolution capability of the proposed method further underscores its potential for resolving closely spaced signals, achieving a resolution accuracy of 98.5%. This exceeds the capabilities reported by Pal and Vaidyanathan [4] for nested arrays and by Candes and Wakin [5] for compressive sensing approaches. While these studies achieved enhanced resolution in specific array configurations, they often required increased computational resources, unlike the proposed method, which maintained computational efficiency.

The computational efficiency of the eigen-method, with an average processing time of 0.43 seconds, makes it suitable for real-time applications. This efficiency surpasses the results of Liu and Zhang [6], who employed deep learning-based localization methods that demonstrated comparable accuracy but required higher computational resources. The current method bridges this gap by achieving both high accuracy and low processing time, emphasizing its practicality for applications in radar and wireless communication systems.

Despite the strengths of the proposed method, some limitations should be addressed. The method's performance was evaluated primarily under controlled simulation and experimental conditions. Future studies should investigate its scalability in more dynamic and multi-path environments, as explored in studies by Capon [7] and Li and Stoica [8]. Additionally, while the eigen-method excels in compact setups, its performance in larger array configurations remains underexplored, warranting further comparative analysis.

In conclusion, this study confirms the efficacy of the eigen-method-based approach for joint frequency and AoA estimation in compact antenna arrays. The method demonstrates clear advantages over traditional techniques, particularly in low SNR conditions, making it a promising solution for real-time signal processing in constrained setups. By building on past research and addressing its limitations, this approach has the potential to advance the field of array signal processing significantly.

Conclusion

The findings of this study underscore the significant advancements achieved through the eigen-method-based approach for joint frequency and angle-of-arrival (AoA) estimation in compact two-element antenna arrays. By leveraging signal subspace decomposition, this method addresses the limitations of conventional techniques like MUSIC and ESPRIT, particularly in scenarios characterized by low signal-to-noise ratios (SNRs) and limited spatial diversity. The proposed method not only demonstrates superior estimation accuracy but also excels in resolution capability and computational efficiency, making it a robust solution for modern signal processing applications. Specifically, the method reduced the mean squared error (MSE) in frequency estimation by up to 48% and AoA estimation by up to 41% compared to MUSIC, highlighting its robustness under adverse conditions. Additionally, it achieved a resolution accuracy of 98.5%, outperforming traditional algorithms, and maintained a low computational overhead with an average processing time of 0.43 seconds, which is critical for real-time applications.

From a practical perspective, these findings offer a transformative approach to designing communication and radar systems, particularly in space-constrained environments such as unmanned aerial vehicles, portable communication devices, and compact radar systems. The robustness of the eigen-method under low SNR conditions makes it particularly valuable for applications in urban and cluttered environments where multipath propagation and noise are prevalent. Moreover, its computational efficiency positions it as an ideal candidate for resource-constrained systems, such as Internet of Things (IoT) devices, which require minimal processing power without compromising accuracy.

Based on the research findings, several practical recommendations are proposed to enhance the implementation and adoption of this technique. First, system designers should consider integrating the eigen-method into software-defined radio platforms for flexible deployment in various frequency bands and signal environments. This integration would allow for dynamic adaptation to changing signal conditions, ensuring optimal performance in diverse scenarios. Second, the method should be further optimized for scalability to support larger antenna arrays and more complex signal environments. While the current study focuses on two-element arrays, extending its application to multi-element configurations could enhance its utility in advanced systems such as massive MIMO and 5G networks. Third, future developments should explore hardware-level implementation of the eigen-method using field-programmable gate arrays (FPGAs) or application-specific integrated circuits (ASICs). Such hardware implementations would further reduce latency and improve energy efficiency, making the method suitable for high-throughput applications.

Additionally, collaboration with industries specializing in signal processing and wireless communications is recommended to facilitate the practical deployment of the method. Joint ventures between academia and industry can lead to the development of optimized solutions tailored to specific applications, such as maritime navigation, vehicular communications, and disaster response systems. These collaborations can also ensure that the algorithm meets regulatory standards and interoperability requirements for various regions and industries.

Finally, this study highlights the need for further research into dynamic and multipath environments to validate the method's performance in real-world scenarios. Expanding the algorithm to account for mobility, such as in vehicular or drone-based platforms, could significantly enhance its applicability. By addressing these practical aspects, the eigen-method-based approach has the potential to redefine the landscape of joint frequency and AoA estimation, contributing to advancements in radar, navigation, and communication technologies.

The proposed eigen-method provides a powerful, efficient, and accurate solution for signal processing in compact antenna arrays. Its practical implications extend to a wide range of applications, offering enhanced performance, reduced complexity, and scalability. By integrating the proposed recommendations into future developments, this method can pave the way for innovative and reliable systems that meet the demands of modern technology.

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