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Superconductivity adding semiconductor structures and a topological insulator

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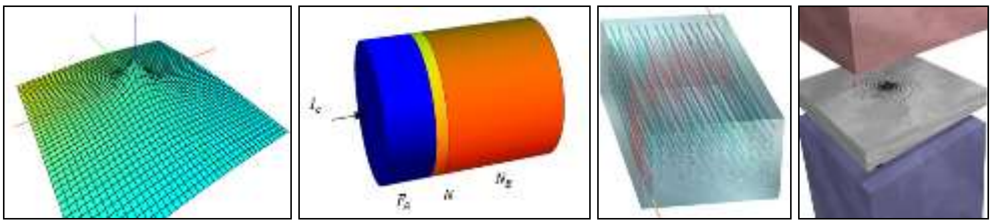
Abstract
From the Majorana bound states on a Kitaev model and the periodic table of topological insulators is designed and developed a superconductor model adding a semiconductor structure that optimizes the quantum spin effects that are determined by proximity effects devices through of TBT-nano transformations. A special topological insulator of magnetic type is introduced incorporated to the semiconductor doing increase the flow of Majorana Fermions into the nano-structure space.

Keywords: Majorana fermions, majorana bound states, semi-conductor, superconductor, topological insulator

1. Introduction

The technological advantages that can appear incorporating a semiconductor element in superconductor devices could has diverse electrical or magnetic qualities that can be used to diverse process of same superconductivity, or the usual conductivity. Obviously also will affect thermal conditions. However arises superconductivity in semiconductors by doping with valence jump metal. The semiconductor element that must added must be metals or metal alloys, in this respect we can see the complete study realized by De Gennes, P. G [1]. The problem of the non-metallic semiconductors is their lack of malleability and these are not ductile, which results a problem when we search a combination of semiconductors with superconductors.



A B C D

Fig 1: A. 2-dimensional surface Model to describe changes in excess carriers being generated to start of increasing light intensity at the center of an intrinsic semiconductor bar. Electrons have higher constant diffusion than holes leading to fewer excess electrons at the center as compared to holes. B. F|N|N theoretical model for Peltier effect simulation when is applying a charge current I_C. Micro-guides in a dielectric material can design and conform an advanced transducer. Vortex states or effects of bound states in the vortex core predicted by N.B. Kopnin; M.M. Salomaa [2] and Majorana fermions, in the studies realized on chiral superconductors by Volovik, G. E [3], or the breaking of parity and time-reversal symmetries realized by N. Read; D., and Green [4], or also the research realized on the manifestation of Majorana fermions in the interfaces states on SC-CM by J. Linder and A. Sudbø [5][1] in spin-orbit coupled semiconductor-superconductor in hybrid structures to superconductivity studies [5].
Likewise, to the superconducting interaction we require the metal alloys semiconductors or even as pure elements as Germanium. An exceptional situation we have with the alloy of germanium telluride. We can see the experiments on GeTe and SnTe on band structure of GeTe and SnTe realized by R. Tsu; *et al.* [6], where the doping is obtained with valence-skipping indium. Likewise, the materials-design research is necessary. Also the superconductivity on a charge diet (when the superconductor material is submitted to a magnetic field or thermal special variations and conditions), can varies the behavior of a superconductor.

[1] A quantum vortex in some superconductors or super-fluids (plasmas or MHD-fluids) can trap mid-gap states, thus is one source of Majorana bound states, newly remit us to F. Bulnes, and S. Humeini [7].

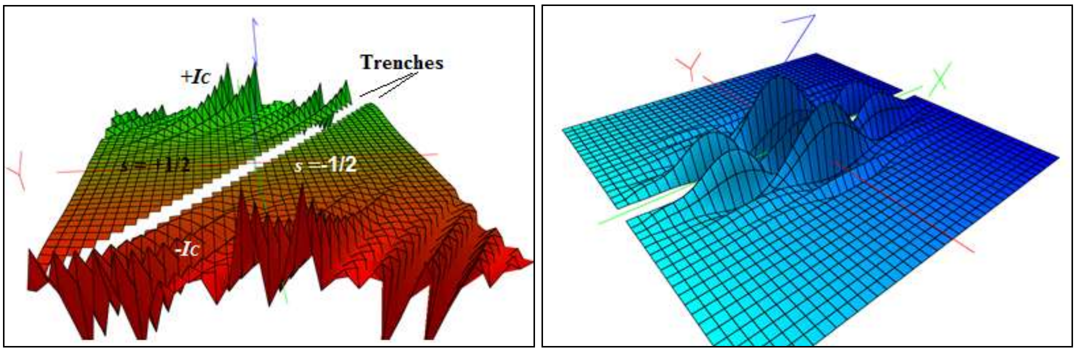


Fig 2: A. We can see superconducting surfaces that can be controlled for harmonics in the junction through their current-phase relation. The polarization (generated from magnetic insulator) of these produce a little potential, having a spintronic effect, which can be used to control the process. The values +1 and -1 are values of scale from the current range, in the bordering of the insulator (generated from semi-conductor). The spins are annulled in trenches in the superconducting process. B. Cyclotron sequence undulations very closer of the chiral edge mode channel. The space between the superconducting surfaces that appears, is the weak link that can consist of a thin insulating barrier, where curvature energy in Hall-spintronics developments are designed from the Berry phase and its curvature, research realized by F. Bulnes, and S. Humeini [7]

However how can establish an interface between superconductor and semiconductor of such luck that we have a good reception and doping of charges, a charge diet by dots in a control through a magnetic insulator, likewise, Cooper pairs are injected from a superconductor into a semiconductor by a process known as reverse Andreev reflection. In a natural way, this mechanism is highly efficient when a good contact between the two materials achieved exists.

The Shockley states at the end points of superconducting wires or line defects are an alternative of source purely electrical, which happens to unpaired Majorana fermions in quantum wires. This was demonstrated by Kitaev, A. Y [8]. A totally different source uses the fractional quantum Hall effect as a substitute for the superconductor, where this was studied too by N. Read; D. Green [4].

One of the goals on this semiconductor disposition is can control the superconductivity using magnetic insulators. Here has high relevance the research realized by L. Fu; C. L. Kane [9] on the behavior that appear in the Majorana fermions when is introduced topological insulator. From these combinations in the corresponding sandwich we obtain the best control of a superconductor, the possibility to obtain a superconducting plus, because we can enounce the following lemma.

Lemma 1. 1. A semi-conductor can be modified to be superconductor. But not vice versa.

Proof. Some materials such that diamonds, alloys with high resistivity $C \to 0.$ and special materials (superconductor polymers) with low resistivity can achieve the superconductor state. However in some materials will be precise add some physical conditions, as doping and low temperature to that achieve the superconductivity state. Now we suppose that a superconductor is a semiconductor. Then this should has holes to be occupied in doping. Indeed, there is holes superconductivity. The hole superconductivity is researched with high precision by J.E. Hirsch [10]. Then the superconductivity state stills being; of fact this defines a fundamental mechanism for superconductivity in itself that arises from the interaction of a hole with the outer electrons in atoms with closer filled shells. The conductivity stills being $\sigma \to \infty.$ which contradicts that the superconductor can be semiconductor. From a point of view experimentally, also exists a very little energy trenches in semiconductors. In semiconductors, we have that the forbidden gap between valence band and conduction band is very small. It has a forbidden gap of about 1 electron volt (*eV*). Then never a superconductor will be semiconductor as such. Only under physical conditions some elements as Germanium can to host superconductivity. But as intrinsic elements no.

2. Hamiltonian of Superconductor-Semiconductor-Insulator Interaction

We must consider all referent to periodic Z2-table for topological insulators and superconductors (see the table 1). The contribution realized by L. Fu; C. L. Kane [9], S. Humeini [11], and C. Nayak, S. Simon, A. Stern, M. Freedman, and S. Das Sarma [12][2] are relevant and with the additional appreciation on magnetic insulators that interact with semi-conductors with behavior of superconductors or a semiconductor with a behavior of a normal conductor that with adequate insulator (of magnetic type) can obtain a partly filled energy band which can be willing to the superconducting process adding the current.

Pure semiconductors are not superconductors. The superconducting condition is achieved when the doping is used. Doping occurs when an element in the semiconductor is replaced by a small percentage of an element in another column of the periodic table, or when a partially ionic semiconductor is prepared with a deficiency of one of the elements in the compound, for example in the R. M. Lutchyn, J. D. Sau, and S. Das Sarma's heterostructures [13]. As was established in the lemma 1. 1, a semi-conductor can be modified to be superconductor.

The fundamental idea of use of a semiconductor is control the electrical current density $J,$ which comes from the superconductor decoupling the Cooper pairs, we recommend newly see the work by F. Bulnes, S. Humeini [7]. This mechanism realizes a doping of semiconductor which choosing a topological insulator of magnetic type, can do to appear the Rashba Effect and Zeeman splitting,

[2] In the table are considered the Bloch-BdG Hamiltonians satisfy the transformations laws (anti-unitary symmetries).

phenomena considered by J. D. Sau, R. M. Lutchyn, S. Tewari, and S. Das Sarma [14], in superconductor heterostructures, the same research group when consider Non-Abelian quantum order in spin-orbit coupled semiconductors [15], and the study realized by T. D. Stanescu, R. M. Lutchyn, and S. Das Sarma [16] in Majorana fermions in semiconductor nanowires. The Rashba effect plays a crucial role in exotic fields of physics such as the found for the Majorana fermions in the semiconductor-superconductor interfaces and the interaction of super-cold atomic Bose and Fermi gases, see the fundamental work realized by L. Cooper [17].

Table 1: Classification periodic table to topological insulators and superconductors.

	Symmetry			d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\boxed{\mathbb{Z}}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\boxed{\mathbb{Z}_2}$	$\boxed{\mathbb{Z}}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\boxed{\mathbb{Z}_2}$	$\boxed{\mathbb{Z}_2}$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Altland-Zirnbauer Random Matrix Classes

Complex K-theory

Real K-theory

Bott Periodicity d→d+8

In the case of introduce a semiconductor layer between superconductor and insulator is obtained a third Hamiltonian block of states which we can describe using; see J. Linder and A. Sudbø [5], the current values in the polarization obtained, for example, with a magnetic insulator (we have the $2\mathrm{D}\,\mathbb{Z}-$ AII-D matrix block from the table 1).

3. Results

Lemma 3.1: We consider a device SC-SM-I, with proportional filtrations regularity corresponding to the two superconductor and semiconductor sides (such as the Josephson junction (*Josephson tri-juction*)) $E_1\psi_1$, and $E_2\psi_2$, and the corresponding change regularity of insulator $A\psi_3$, have solutions in a space [3] (fermionic Fock space proposed by I. Verkelov, R. Goborov, F. Bulnes) [18]:

$$Nano_{SC-SM-I} = s_1\mathcal{T}(\mathrm{M})\otimes s_2\mathcal{T}(\mathrm{M})\otimes s_3\mathcal{T}(\mathrm{M}), \qquad (1)$$

where M is the material space (which we consider as alloy), and T, is the TBT-nano transformation of the space (that is to say, the Bogolobiub transformation). s_1, s_2, and s_3, are spins ordered in the two fundamental categories $\downarrow\uparrow$. The spin splitting in the superconductor alone cannot induce a topological transition in the semiconductor. The spin splitting in the superconductor requires a magnetic insulator (see the fundamental woyk by A. Kitaev [19]). Then the proximity effects device, as discussed before and considered in the study realized by L. Fu; C. L. Kane [9]; considers the 3-body operator which produces the induced proximity given by the term

$$\Gamma_{3^+} = \Gamma_{3^-} = \int d\mathbf{r}(w_3(\mathbf{r})\Phi(\mathbf{r}) + z_3(\mathbf{r})\Phi(\mathbf{r})), \qquad (2)$$

$$\Delta_S\psi_\uparrow{}^\dagger\psi_\downarrow{}^\dagger + \Delta_S{}^*\psi_\downarrow\psi + \Delta_S, \qquad (3)$$

Proof. We consider the Hamiltonian as

$$H = i\hbar\upsilon_F(\gamma_L\partial_x\gamma_L - \gamma_R\partial_x\gamma_R) + i\Delta\cos(\phi/2)\gamma_L\gamma_R, \qquad (4)$$

and we express the derivatives terms $\gamma_L\partial_x\gamma_L - \gamma_R\partial_x\gamma_R$, as wave functions inside a bracket. The relations between these wave functions are multi-linear and the total Hamiltonian satisfies the relation:

$$H = \sum E_n\Gamma_n^\dagger\Gamma_n, \qquad n = 3 \qquad (5)$$

This concludes the demonstration with a little work on the evolved operators and their properties.

Due to the lemma 3. 1, we can consider the design of a mixture of Josephson junction type of three elements superconductor-semiconductor-topological insulator, and obtain the Majorana Fermions flow as mentioned by Wilczek, F [20]4 in each interphase and represented by (3).

If we called superconducting part the junction SC-SM, we can affirm considering the mentioned before on the control role of magnetic insulator.

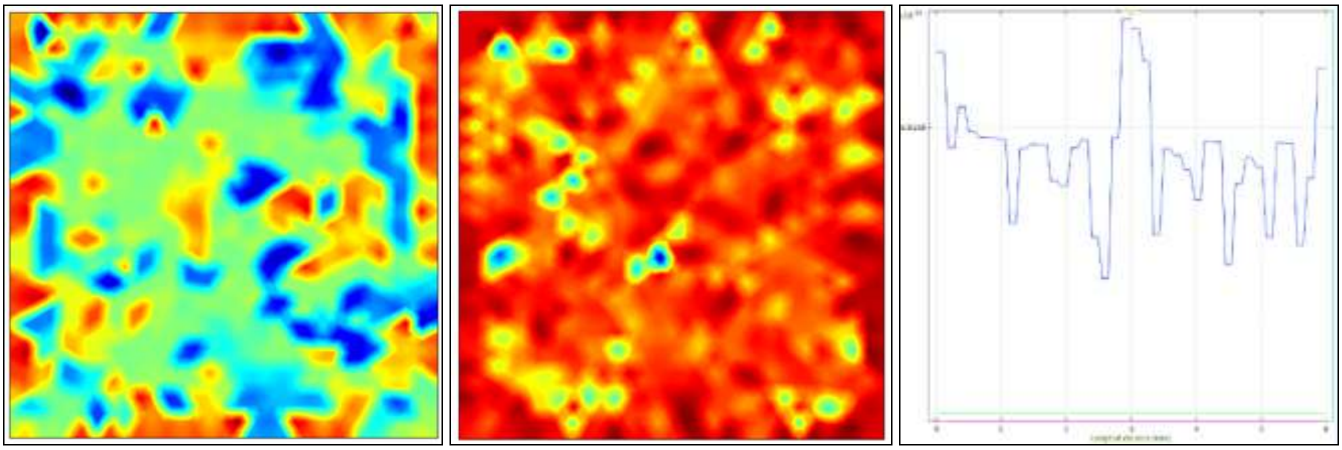
Lemma (F. Bulnes) 3.2: The external Voltage obtained outside the insulator zone controls the total superconducting energy in a Josephson Effect.

Proof. See F. Bulnes, and S. Humeini [7].

We can use the lemma 3. 1, to design an intersidereal magnetic field detector (which could be useful in the torsion detection with additional devices), considering the super-currents existing in a SQUID as quantum macroscopic interference of large scope device. In the Universe the super-currents are aroused from the electromagnetic plasma in the star and galaxies nucleus.

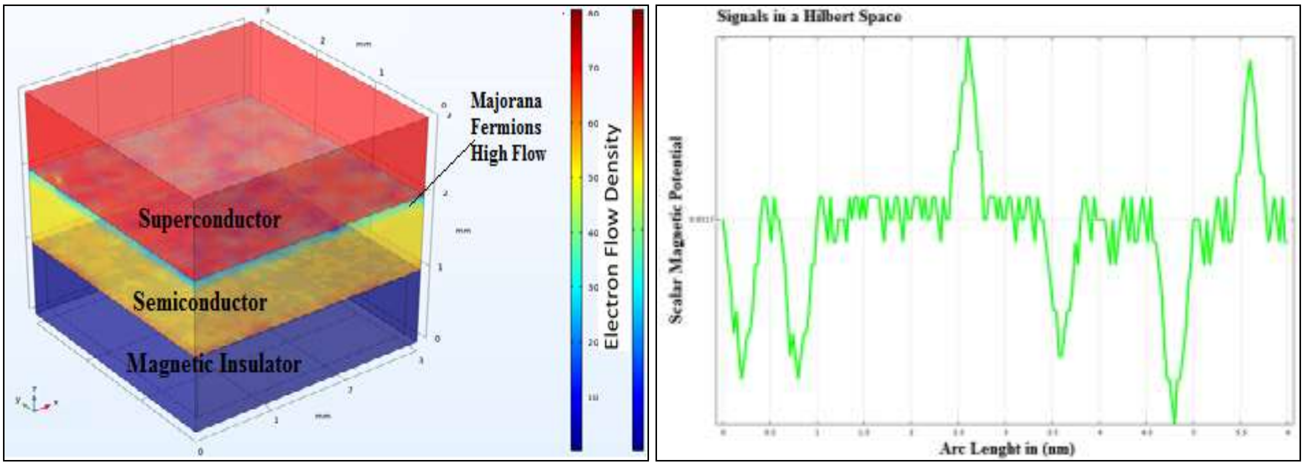
3 This space is of the Nano transformations (see the mathematical theory of Nanotechnology). In particular the TBT-Nano is such transformation.

4 Wilczek, F, Nobel Prize by his research on insulators. He says that the insulators are the windows of the Universe in its more deep aspect.



A B C

Fig 3: A. Concentration of holes in the superconductor-semiconductor patron. B. Electron flow in semiconductor-topological insulator patron. C. Energy quasi-levels of Fermi to electrons.



A B

Fig 4: A. Three junction SC-SM-I. The Majorana Fermions high flow is generated due the presence of the magnetic insulator. B. Scalar magnetic potential signals consigned in a Hilbert space interphase. This is measured from the magnetic insulator interacting with the superconductor-semiconductor junction (these simulations realized by Victor A. Sánchez).

4. Conclusions

The Josephson effect is fundamental in the structure design of control of the superconducting current considering some components as topological insulators where always must be present the momentum conservation law due to that superconducting current is equal to Cooper pairs current and these must be break if we want find a fine effect of superconducting current, as for example in a semi-conductor or variation of weak magnetic field, or even the design of the micro-current hyper-sensor in insulator prescience with quantum actions or effects of quantum level.

A form of topological components handling in a quantum device could be through of Majorana bound states. Here we recommend various studies realized to the Kitaev's model realized for example M. Burrello [21], some quantum multi-Physics simulations of the Hall Effect in superconductors to the Kitaev's model by Mahmoud, J [22], the Kitaev himself additional research in Kitaev, A. Y [23], on the non-abelions in quantum Hall effect by G. Moore; N. Read [24], and finally the realized study by Lee, H.-Y. *et al.*, [25], on Kitaev´s magnetons; where these create the wave function through the spinor frame that naturally can be measured in energy spaces (Hilbert spaces) to organized transformations (Fock spaces), such as happens in a Fermionic Fock space (see again I. Verkelov, R. Goborov, F. Bulnes [18]). The idea of create a periodic table to insulators and superconducting such as given in the table 1, where it has the treatise by B.A. Bernevig and T.L. Hughes [26]; is can to create through of matrix blocks the superposing wave function (spinors) obtained by ortho-normality in the decomposition of the Hilbert space solution (with sub-blocks defined by the state matrices in sub-spaces) as:

$$\mathcal{H} = \mathcal{H}_1 \oplus \ldots \oplus \mathcal{H}_n, \tag{6}$$

Having the mean field of Hamiltonian given by $H = \frac{1}{2}\sum(\Psi^\dagger, \Psi) H_{BdG} \begin{pmatrix} \Psi \\ \Psi^\dagger \end{pmatrix}$, where we have the two Majorana states $\Psi^\dagger, \Psi$) and the Kitaev models can to appear; can be seen the work by M. Burrello [21]. The correlation between components, for example, SC-SM-I, can be given by relation between path integrals such as could be the relations of Majorana bound states between particles where in some cases there is particle (full state) or there not is particle (hole state). Finally we can affirm that the junction of these three elements SC-SM-I, or even in a special alloy or mixture will can be obtained possibly a hybrid mixture with superconductor behavior in ambient temperature and standard pressure conditions. The lemma 3. 1, suggest it, considering the TBT-Nano transformation of the space (1).

5. Technical Notation

C – Capacity

$\downarrow\uparrow$ – Spin pair.

$\psi_i\,(i=1,2,\ldots,n)$ – ith – wave function.

$\Gamma^{\dagger}_{E=0}=\Gamma_{E=0}\equiv\gamma$ – Majorana Fermion states

$\Gamma_{n^+}=\Gamma_{n^-}=\int d\mathbf{r}(u_n(\mathbf{r})\Psi(\mathbf{r})+\upsilon_n(\mathbf{r})\Psi(\mathbf{r}))$ – n body operators

H – Hamiltonian.

$\bigotimes_n s_{\iota}\mathcal{T}(\mathrm{M})$ – Tensor product space of the TBT-nano transformations on the material M. This conservers the Hilbert and Fock topologies spaces.

$s_{\iota}\,(\mathrm{i}=1,2,\ldots,n)$ – ith – spin.

σ – Conductivity.

$\otimes$ – Tensor products of elements of a vector space or module. Here the vector spaces or modules are Hilbert sub-spaces.

$\mathcal{H}$ – Hilbert space. This is a topological vector space of infinite dimension, which has a 2-norm to measure the length of its elements (which are functions or signals). The square of this norm defines a quadratic form which defines from a physical point a view, energy of a signal.

$\oplus$ – Direct sum of vector spaces even modules. $A\oplus B$, means that $A+B$, satisfies $A\cap B=\{0\}$. When the direct sum is of vector spaces then defines orthogonally among its elements.

6. Nomenclature

SQUID- SQUID whose acronym mean "Superconducting Quantum Interference Device", which is a magnetometer very sensitive used to measure extremely weak magnetic fields. This is based on superconducting loops (topologies) containing Josephson junctions.

Fermionic Fock Space- A Fermionic Fock space is an algebraic construction used in quantum mechanics to construct the quantum states space (to fermions) of a variable or unknown number of identical particles from a single particle Hilbert space $\mathcal{H}$. Formally, the Fermionic Fock space is (the Hilbert space completion) the direct sum of the symmetric or anti-symmetric tensors in the tensor powers of a single-particle Hilbert space $\mathcal{H}$,

$$F_{-\nu}(\mathcal{H})=\bigoplus_{n=1}^{\infty}Sym\mathcal{H}^{\otimes n},$$

This space has a standard basis, which can be indexed by a variety of objects.

$2\mathrm{D}\,\mathbb{Z}$ – These 2d and 3d topological insulators are characterized by a Z2-valued invariant, and they are stable under small perturbations by impurities or defects, so such materials are called Z2 topological insulators.

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